Dynamic Pricing of Relocating Resources in Large Networks

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David Brown
Chen Chen

February 2021

Motivation

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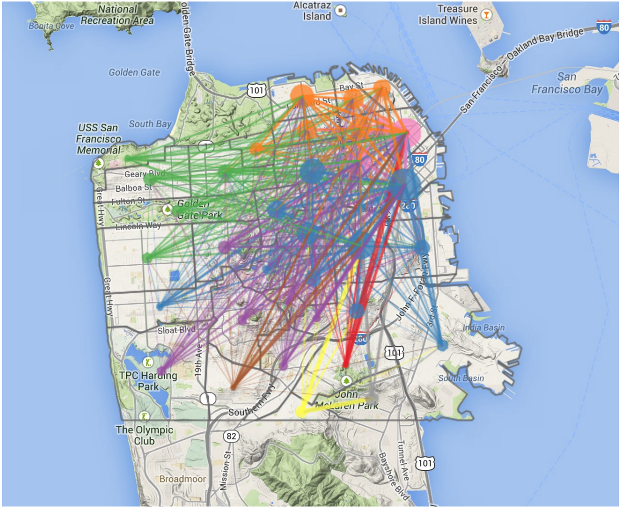

Ride flow in San Francisco Source: #UberData

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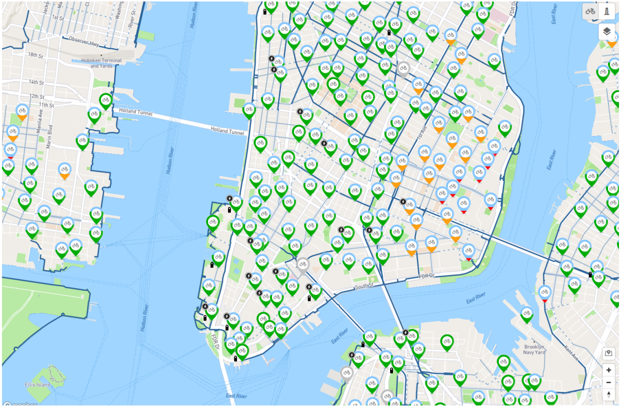

Bike sharing in NYC
Source: citibikenyc.com

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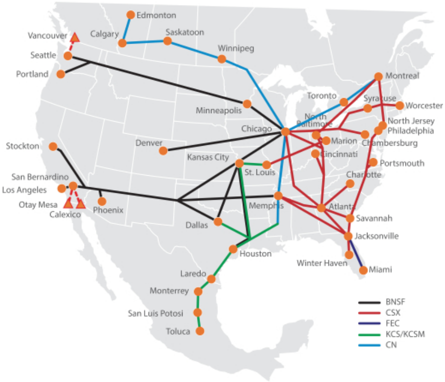

Logistics networks
Source: Schneider

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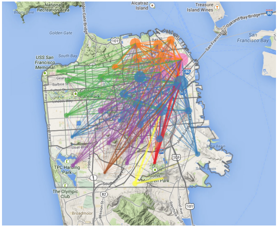

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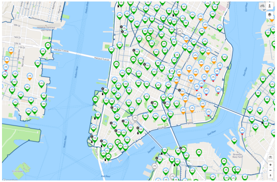

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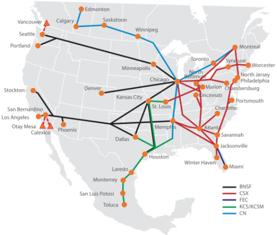

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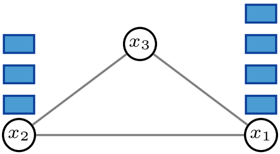
- The spatiotemporal distribution of resources can be controlled through **pricing**.
- The underlying networks may be **large** and often contain some **central** locations of key importance.
- **Challenge:** optimal dynamic pricing policies may be very difficult to compute.

Research Question:

Can we design "simple" dynamic pricing policies that perform well in these problems?

Problem formulation

m resources distributed over n locations; x_i = number of resources at i



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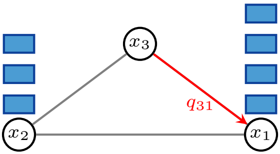
In each period:

1. A customer requests (i, j) with probability q_{ij}
   - Private willingness-to-pay$\sim F_{ij}(p) = \text{Prob}\{\text{value}_{ij} \geq p\}$, independent



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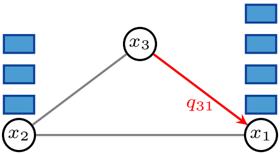

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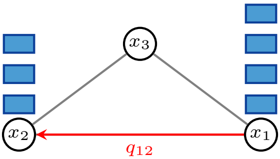

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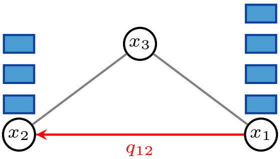

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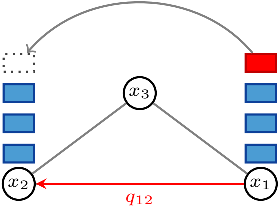
   - Platform selects price p
   - With probability $F_{ij}(p)$, request is accepted:
     $x_i \to x_i - 1$ and $x_j \to x_j + 1$ and revenue p collected



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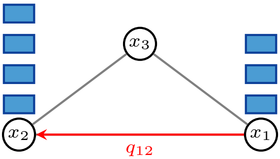

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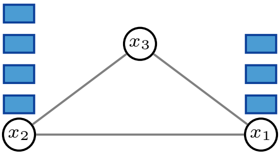

Problem: *find a dynamic pricing policy that maximizes average revenue.*

Overview

- **Goal:** find "simple" policies and establish bounds on suboptimality.
- **Large supply regime:** locations n fixed, resources $m \to \infty$.
  - Problem is $\approx$ deterministic and *fluid relaxations* perform well:
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  - Appropriate for dense urban areas with high demand/supply per location.

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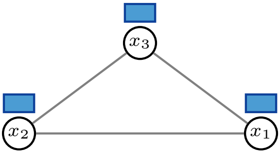

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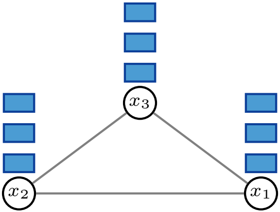

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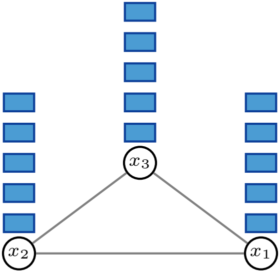

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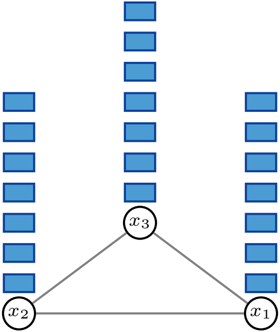

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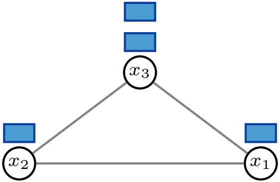

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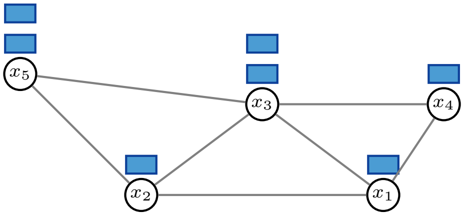

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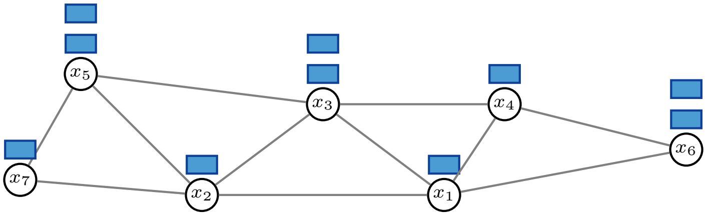

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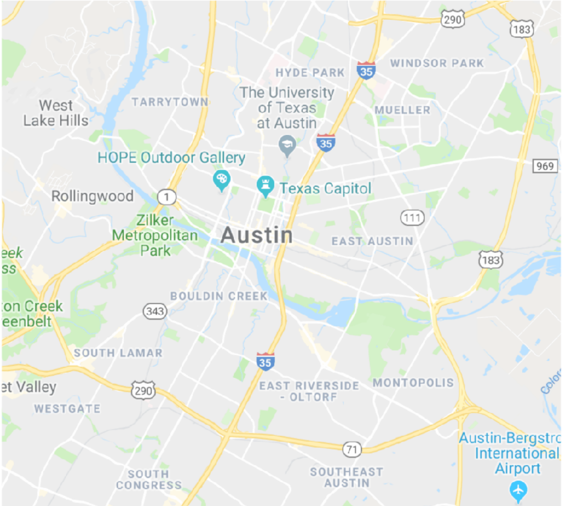
Main result: develop dynamic pricing policies and performance bounds based on *Lagrangian relaxations* for networks with a "hub-and-spoke" structure.

$\Longrightarrow$ Asymptotic optimality of a dynamic policy in the large network regime.

High-level idea



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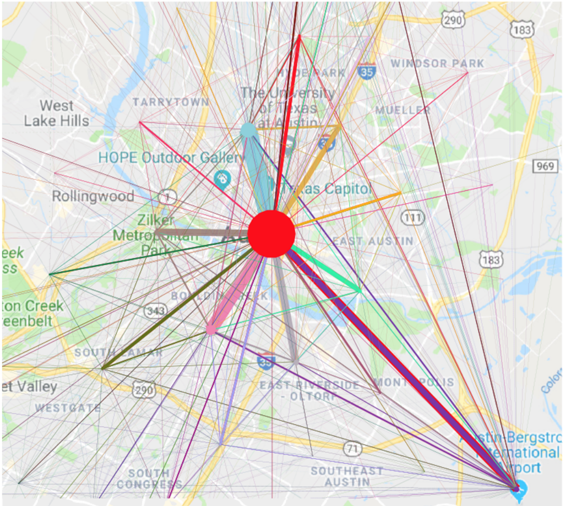
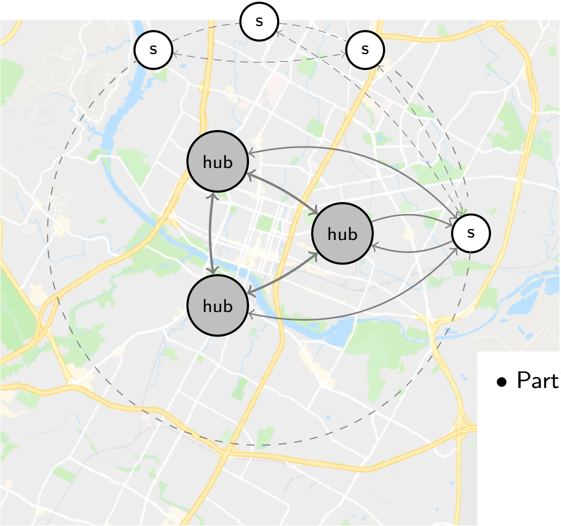


Data: RideAustin

High-level idea



- Partition the city into hubs and spokes

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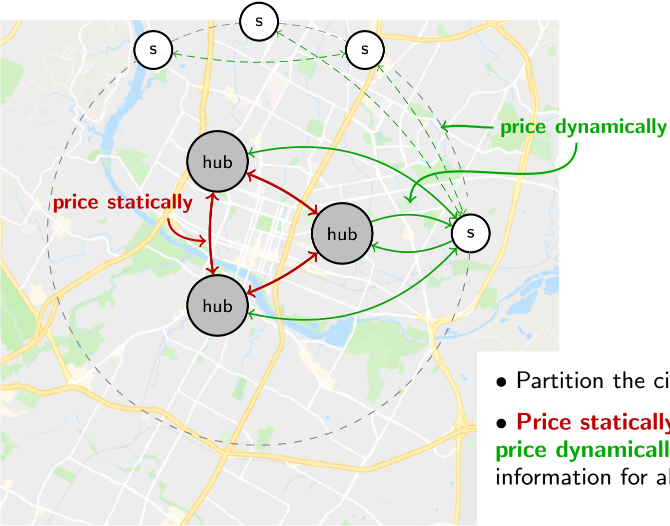

- Partition the city into hubs and spokes
- **Price statically** between hubs and **price dynamically** based on “local” information for all other requests

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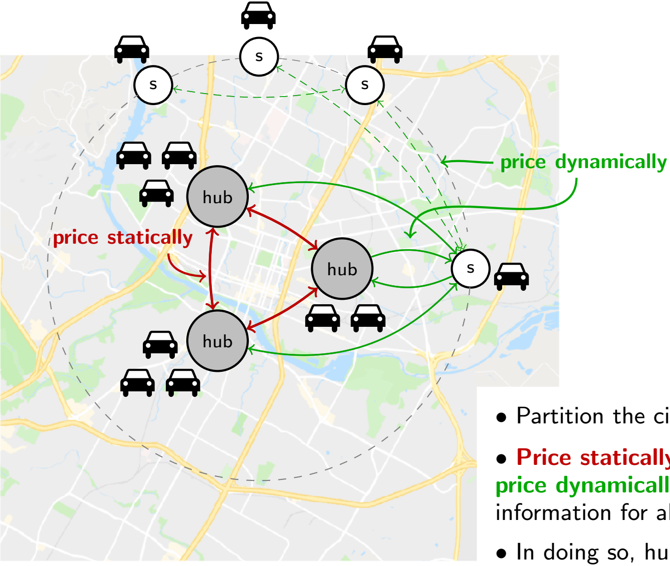


- Partition the city into hubs and spokes
- **Price statically** between hubs and **price dynamically** based on "local" information for all other requests
- In doing so, hubs pool resources and we maintain a small number of resources at each spoke (on average)

Literature review

Shared vehicle systems:

- Fluid relaxations: Waserhole and Jost (2016), Banerjee et al. (2016)
  $\Longrightarrow$ Show fluid-policy is within a factor of $\frac{m}{m+n-1}$ of optimal.
- Assignment and relocation of resources: Braverman et al. (2016), Ozkan and Ward (2016), Banerjee et al. (2018), Kanoria and Qian (2020), Benjaafar et al. (2018)
- Strategic drivers: Bimpikis et al. (2019), Besbes et al. (2018), Afèche et al. (2018)

Logistics and transportation networks:

- ADP for capacity control: Adelman (2007)
- Hub-and-spoke networks: Du and Hall (1997), Pirkul and Schilling (1998), Song and Carter (2008)
- Closed queueing networks: Gordon and Newell (1967), George and Xia (2011)

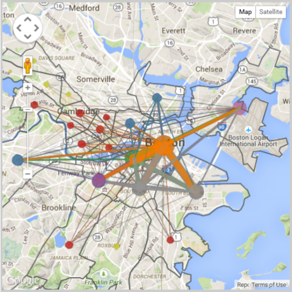
Lagrangian relaxations of weakly coupled stochastic DPs:

- Methodology: Hawkins (2003), Adelman and Mersereau (2008), Bertsimas and Mišić (2017), Brown and Smith (2018)
- Applications:
  - Network revenue management: Topaloglu (2009)
  - Marketing: Bertsimas and Mersereau (2007), Caro and Gallien (2007)
  - Multi-armed bandits: Brown and Zhang (2020)
  - Inventory control: Miao et al (2020)

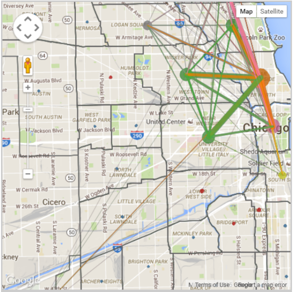
Outline

- **Motivation, problem, and literature review (done)**
- **Hub-and-spoke networks**
  - Lagrangian relaxation: provides an upper bound and a feasible policy
  - Performance analysis and asymptotic optimality
  - Examples
- **More general networks:** build upon methodology and theory above
  - Multiple, interconnected hubs
  - RideAustin Example
- **Conclusions**

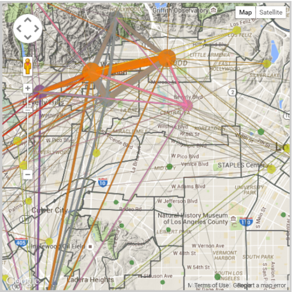
Visualizing flow & hubs in ridesharing



Boston



Chicago



Los Angeles

We can identify networks of "related" neighborhoods that are the "hub" of the city, into and out of which the most people flow.

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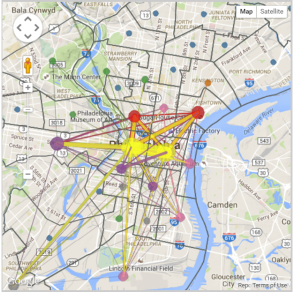
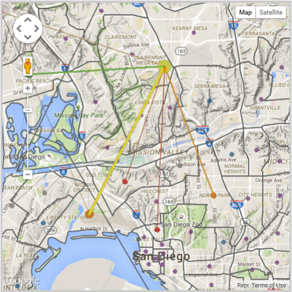
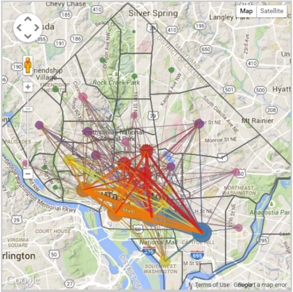


Philadelphia



San Diego



Washington, D.C.

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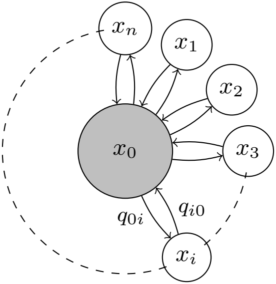
Hub-and-spoke network



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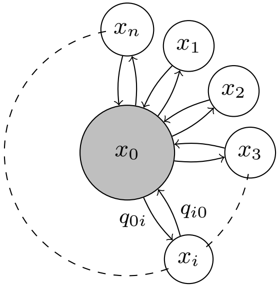
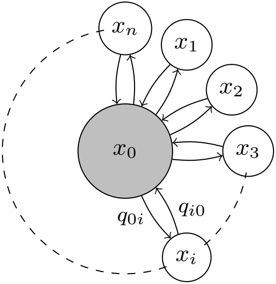


- n spokes, one hub and m resources.
- Continuous-time model with Poisson arrivals
  $\Rightarrow$ consider the embedded discrete-time Markov chain.
- In each period, a request for (i, j) arrives with probability q_{ij}.
- Service provider equivalently selects a demand level $d = F_{ij}(p) \in [0, 1]$.
- One-period expected revenue $r_{ij}(d) = d \cdot F_{ij}^{-1}(d)$, concave in d.
- Relocations are instantaneous.
- Resources only move when fulfilling requests.

Hub-and-spoke network



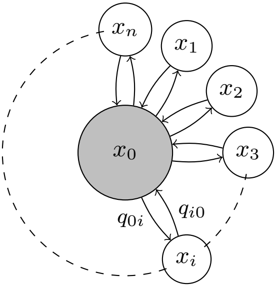
0-1 r.v. whether request at t is $(0, i)$

$$V^{\mathrm{OPT}} = \max_{\pi \in \Pi} \quad \lim_{T\to\infty} \frac{1}{T} \cdot \mathbb{E}\left\{ \sum_{t=1}^{T} \sum_{i=1}^{n} \left(\underbrace{y_{i0,t} \cdot r_{i0}\left(d^{\pi}_{i0,t}\right)}_{\text{Revenue of requests from spoke } i \text{ to hub}} + \underbrace{y_{0i,t} \cdot r_{0i}\left(d^{\pi}_{0i,t}\right)}_{\text{Revenue of requests from hub to spoke } i} \right) \right\}$$

set of feasible policies

s.t. Dynamics of resources .

Hub-and-spoke network



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set of feasible policies

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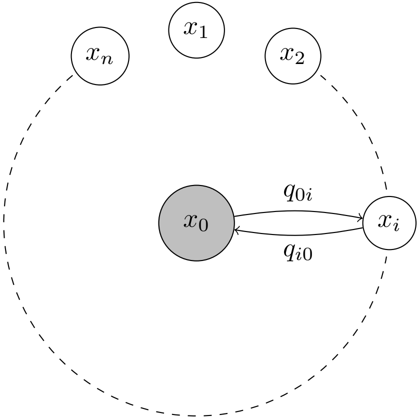
- V^{OPT} is independent of the "initial" state of the system
- Optimal policies depend on the **full** system state $\mathbf{x} \triangleq (x_0, \ldots, x_n)$

Lagrangian relaxation



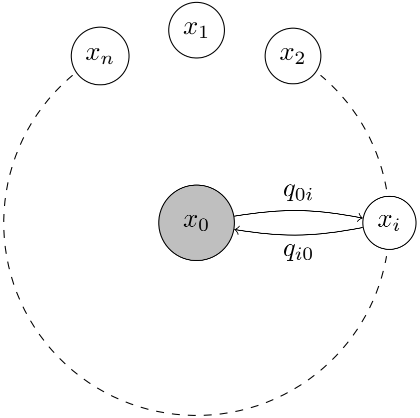
Relax constraint on hub resources:

$x_0 \geq 0$

$$V^{\mathrm{OPT}} = \max_{\pi \in \Pi} \lim_{T \to \infty} \frac{1}{T} \cdot \mathbb{E} \sum_{t=1}^{T} \sum_{i=1}^{n} \left(y_{i0,t} \cdot r_{i0}\left(d^{\pi}_{i0,t}\right) + y_{0i,t} \cdot r_{0i}\left(d^{\pi}_{0i,t}\right) \right)$$

s.t. Dynamics of resources.

Lagrangian relaxation



Relax constraint on hub resources:

$$x_0 \geq 0 \iff \underbrace{m - \sum_{i \in [n]} x_i \geq 0}_{\text{Lagrange mult. } \lambda \geq 0}$$

$$x_0 + \sum_{i \in [n]} x_i = m$$

$$V^{\text{OPT}} = \max_{\pi \in \Pi} \lim_{T \to \infty} \frac{1}{T} \cdot \mathbb{E} \sum_{t=1}^{T} \sum_{i=1}^{n} \left(y_{i0,t} \cdot r_{i0}\left(d^{\pi}_{i0,t}\right) + y_{0i,t} \cdot r_{0i}\left(d^{\pi}_{0i,t}\right) \right)$$

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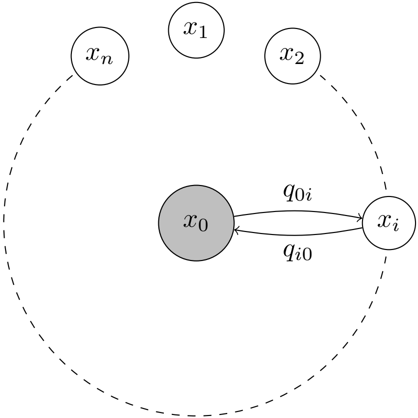

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$$\bar{V}^{\lambda} = \max_{\pi \in \bar{\Pi}} \lim_{T \to \infty} \frac{1}{T} \cdot \mathbb{E} \sum_{t=1}^{T} \sum_{i=1}^{n} \left(y_{i0,t} \cdot r_{i0}\left(d^{\pi}_{i0,t}\right) + y_{0i,t} \cdot r_{0i}\left(d^{\pi}_{0i,t}\right) \right) + \lambda \left(m - \sum_{i=1}^{n} x^{\pi}_{i,t} \right)$$

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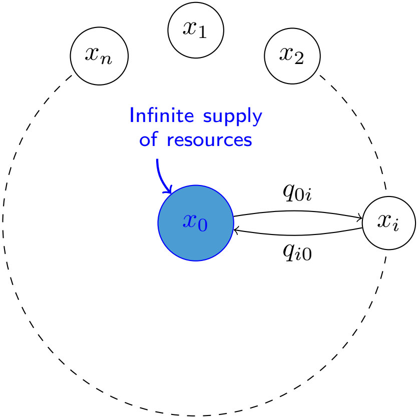
Lagrangian relaxation



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Relaxed problem **decouples** over spokes!

$$\bar{V}^{\lambda} = \lambda m + \sum_{i=1}^{n} \max_{\pi \in \bar{\Pi}} \lim_{T \to \infty} \frac{1}{T} \cdot \mathbb{E} \sum_{t=1}^{T} \left(y_{i0,t} \cdot r_{i0}\left(d^{\pi}_{i0,t}\right) + y_{0i,t} \cdot r_{0i}\left(d^{\pi}_{0i,t}\right) - \lambda x^{\pi}_{i,t} \right)$$

s.t. Dynamics of resources .

Properties of the Lagrangian relaxation

For any dual variable $\lambda \geq 0$:

1. $\bar{V}^{\lambda}$ *is independent of the initial state.*
2. *Weak duality holds, i.e.,* $\bar{V}^{\lambda} \geq V^{\mathrm{OPT}}$.
3. *The Lagrangian relaxation decomposes over spokes:*

$$\bar{V}^{\lambda} = m\lambda + \sum_{i=1}^{n} h_i^{\lambda} \ .$$

$h_i^{\lambda} \triangleq$ optimal average revenue of spoke i with penalty λ

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h_i^{λ} equals the optimal value of a **spoke-specific DP** with $\approx m$ states.

Size of state space:

(a) Full DP: $O(m^n)$

(b) n spoke-specific DPs from Lagrangian relaxation: $O(n \cdot m)$

The spoke problem

Proposition. h_i^λ equals the optimal value of:

$$\max_{\substack{d_i(x,\text{in}),\\ d_i(x,\text{out}),\\ p_i(x)\geq 0}} \quad \underbrace{\sum_{x=0}^{m} p_i(x)\Big[q_{i0}\cdot r_{i0}\Big(d_i(x,\text{out})\Big) + q_{0i}\cdot r_{0i}\Big(d_i(x,\text{in})\Big)\Big]}_{\text{Revenue from requests between hub and spoke } i} - \underbrace{\lambda\cdot\sum_{x=0}^{m} x\cdot p_i(x)}_{\text{Penalty for holding resources at spoke } i}$$

$$\text{s.t.} \quad \sum_{x=0}^{m} p_i(x) = 1,$$

$$p_i(x)\cdot q_{0i}\cdot d_i(x,\text{in}) = p_i(x+1)\cdot q_{i0}\cdot d_i(x+1,\text{out}), \qquad \textbf{(flow balance)}$$

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p_i: stationary distribution of resources at i

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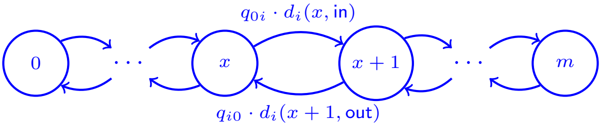

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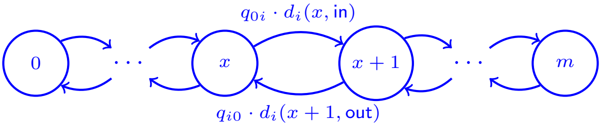
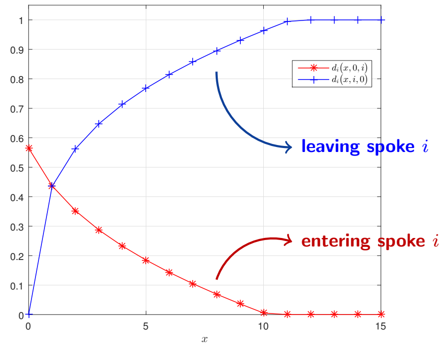
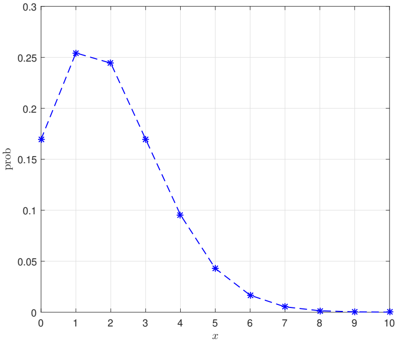


This problem is **non-convex**, but can be formulated as a convex problem over $p_i(x)$.

Structural insights

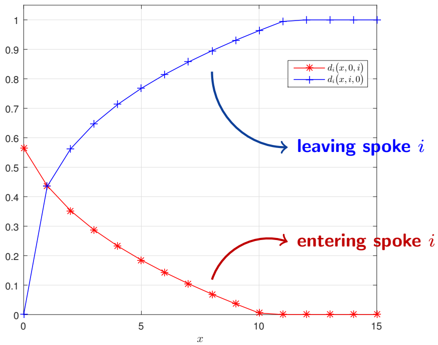


Monotonicity: The *Lagrangian policy* controls $d_i(x, \mathsf{out})$ and $d_i(x, \mathsf{in})$ are increasing and decreasing in x, respectively.



Log-concavity: The distributions of resources in the spokes are discrete log-concave.

Structural insights



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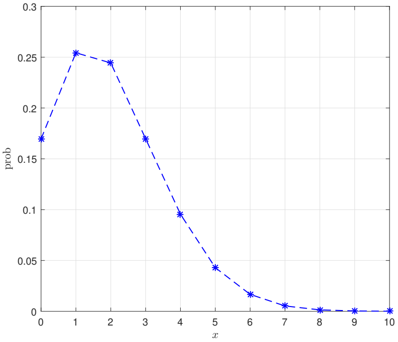

Log-concavity: The distributions of resources in the spokes are discrete log-concave.

The Lagrangian policy can be implemented in the original system (the policy, however, needs to drop requests when the hub is empty).

The Lagrangian dual problem

The Lagrangian dual (*convex*) problem:

$$V^{\mathrm{R}} \triangleq \min_{\lambda \geq 0} \bar{V}^{\lambda}$$

Let λ^* denote an optimal solution. From complementary slackness:

$$\lambda^* \cdot \left(m - \overbrace{\sum_{i=1}^{n} \sum_{x=0}^{m} x \cdot p_i^* (x)}^{\text{total resources at spokes}} \right) = 0\,.$$

optimal stationary distribution at λ^*

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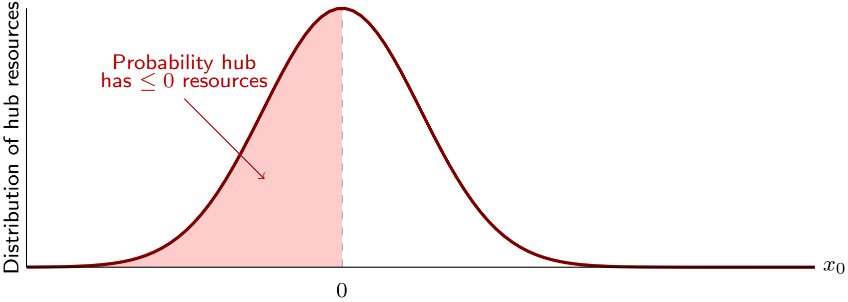

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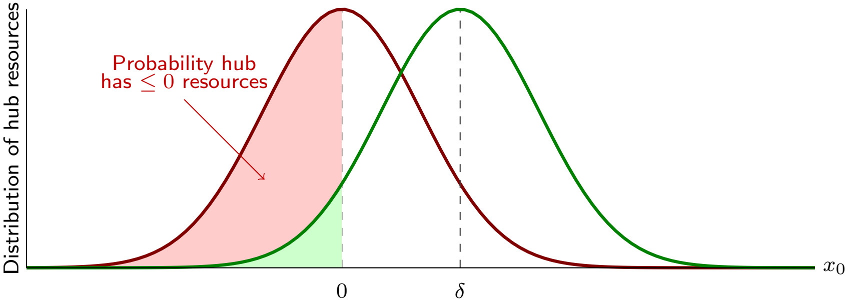

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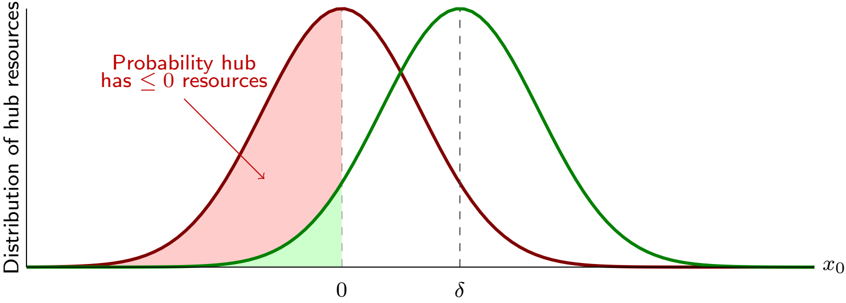
The *perturbed* Lagrangian dual problem:

$$V^{\mathrm{R}}(\delta) \triangleq \min_{\lambda \geq 0} \big\{ \bar{V}^{\lambda} - \delta\, \lambda \big\}$$

Let λ^* denote an optimal solution. From complementary slackness:

$$\lambda^* \cdot \left((m-\delta) - \overbrace{\sum_{i=1}^{n} \sum_{x=0}^{m} x \cdot \underbrace{p_i^*}_{\text{optimal stationary distribution at } \lambda^*}(x)}^{\text{total resources at spokes}} \right) = 0\,.$$



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Will choose $\delta = o(n)$ (later)

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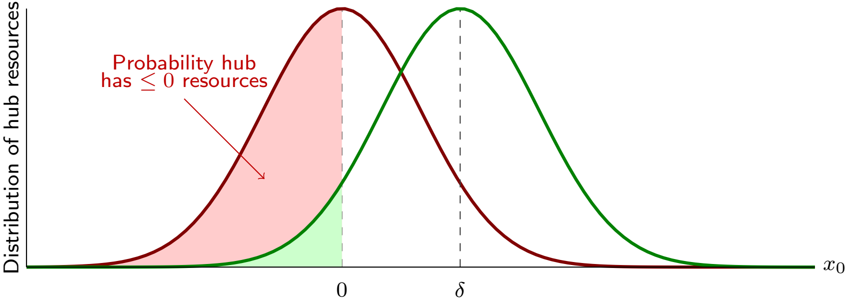

Performance analysis

Goal: $$\underbrace{V^{\pi}(\delta)}_{\text{Lagrangian-based policy}} \leq V^{\mathrm{OPT}} \leq V^{\pi}(\delta) + \underbrace{\mathsf{Error}(n)}_{\xrightarrow[\frac{m}{n}\text{ fixed}]{n\to\infty} 0}$$

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upper bound on derivative of $r(d)$

hub depletion probability with $\pi(\delta)$

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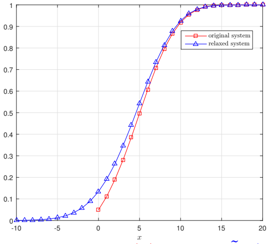
Hub depletion probability

Lemma. For any δ, we have $\underbrace{X_0(\delta)}_{\text{original}} \succeq_{\text{FOSD}} \underbrace{\tilde{X}_0(\delta)}_{\text{relaxed}}$.



Hub CDFs: $X_0(\delta) \succeq_{\text{FOSD}} \tilde{X}_0(\delta)$

Hub depletion probability

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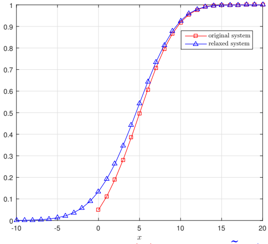

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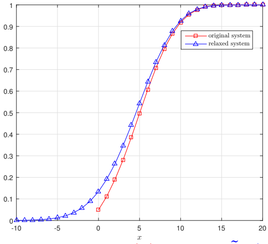

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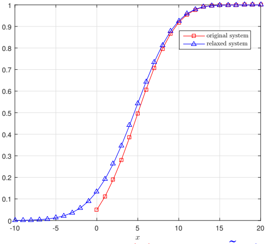

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Proposition. The hub depletion probability satisfies:

independent of n

$$\mathbb{P}[X_0(\delta) = 0] \leq \mathbb{P}[\tilde{X}_0(\delta) \leq 0] \leq e^{-\beta \cdot \frac{\delta^2}{n}}$$

Performance result

Theorem. *The Lagrangian policy $\pi(\delta)$ satisfies*

$$V^{\pi}(\delta) \leq V^{\mathrm{OPT}} \leq V^{\mathrm{R}} \leq V^{\pi}(\delta) + \bar{r} \cdot \frac{\delta}{m-\delta} + (\bar{r} + \bar{\omega}) \cdot e^{-\beta \cdot \frac{\delta^2}{n}} \ .$$

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Moreover, with $\delta = \sqrt{\frac{1}{2\beta} \cdot n \cdot \ln n}$, we have

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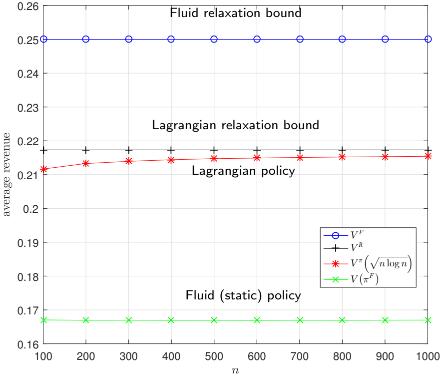
- Policy keeps, on average, $O(\sqrt{n \cdot \ln n})$ resources in the hub and $O(1)$ resources in the spokes.
- Result holds when spokes are asymmetric.

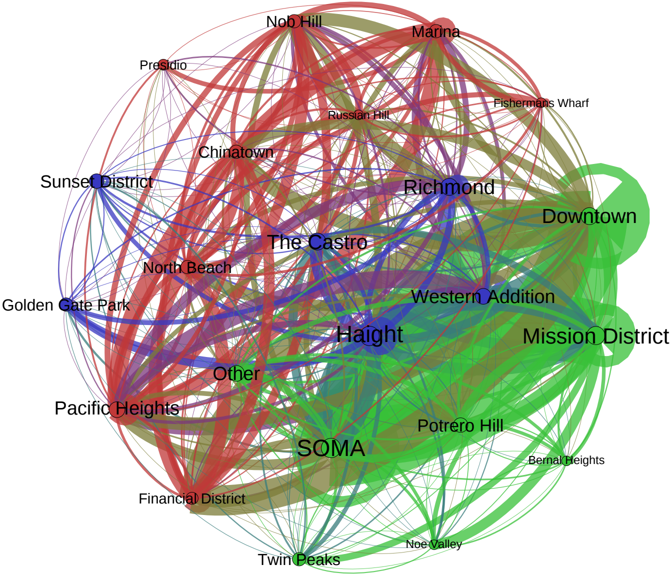
Single hub examples

(a) symmetric spokes with $q_{i0} = q_{0i} = \frac{1}{2n}$; (b) $\frac{m}{n} = 2$; (c) all private values $\sim \mathrm{U}[0,1]$



- Lagrangian policy is asymptotically optimal.
- Fluid policy: performance gap $\big(V^{\mathrm{F}} - V(\pi^{\mathrm{F}})\big)/V(\pi^{\mathrm{F}}) = \frac{2}{3}$.

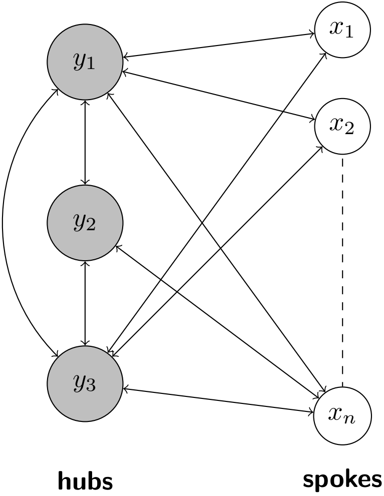
More general networks



Based on Uber GPS data (source: `blogs.mathworks.com`)

Multiple hub networks

$$\overbrace{\sum_{j\in[J]} y_j}^{J\text{ hubs}} + \overbrace{\sum_{i\in[n]} x_i}^{n\text{ spokes}} = m$$

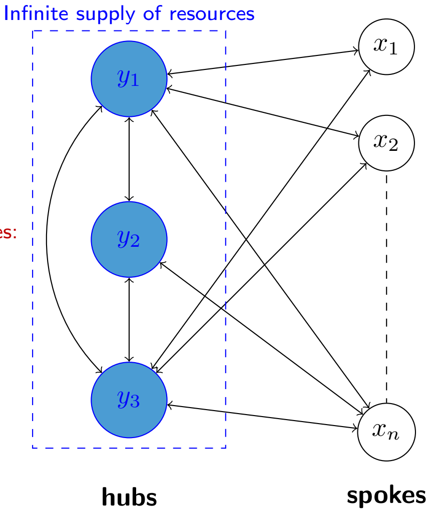


Multiple hub networks

$$\overbrace{\sum_{j\in[J]} y_j}^{J \text{ hubs}} + \overbrace{\sum_{i\in[n]} x_i}^{n \text{ spokes}} = m$$

Relax constraint on hub resources:

$$\sum_{j\in[J]} y_j \geq 0 \Leftrightarrow \underbrace{m - \sum_{i\in[n]} x_i \geq 0}_{\text{Lagrange mult. } \lambda\geq 0}$$



Multiple hub networks

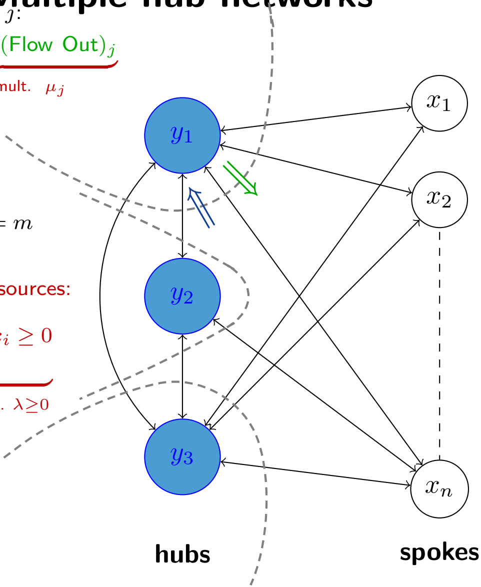
For each hub j:

$$\underbrace{(\text{Flow In})_j = (\text{Flow Out})_j}_{\text{Lagrange mult. } \mu_j}$$

$$\overbrace{\sum_{j\in[J]} y_j}^{J \text{ hubs}} + \overbrace{\sum_{i\in[n]} x_i}^{n \text{ spokes}} = m$$

Relax constraint on hub resources:

$$\sum_{j\in[J]} y_j \geq 0 \Leftrightarrow \underbrace{m - \sum_{i\in[n]} x_i \geq 0}_{\text{Lagrange mult. } \lambda\geq 0}$$



Multiple hub networks

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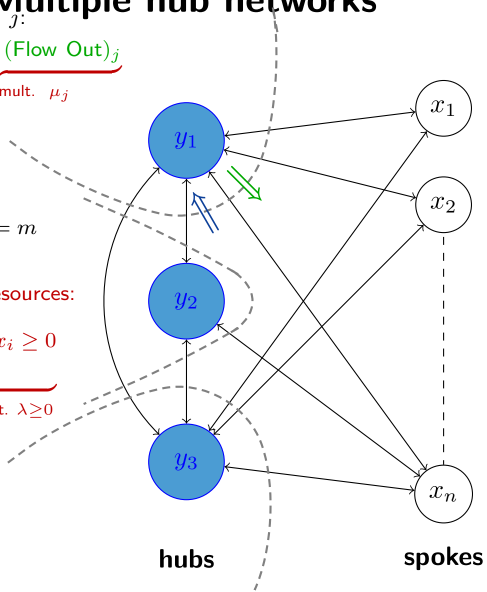

Relaxation **decouples** across spokes and hubs!

Properties of the Lagrangian relaxation

For any $\lambda \geq 0$ and any $\boldsymbol{\mu} \in \mathbb{R}^J$:

1. $\bar{V}^{\lambda,\boldsymbol{\mu}}$ *is independent of the initial state.*
2. *Weak duality holds, i.e.,* $\bar{V}^{\lambda,\boldsymbol{\mu}} \geq V^{\text{OPT}}$.
3. *The Lagrangian relaxation decomposes over spokes and hubs:*

$$\bar{V}^{\lambda,\boldsymbol{\mu}} = m\lambda + \sum_{i\in[n]} \underbrace{h_i^{\lambda,\boldsymbol{\mu}}}_{\text{spoke-specific DP for } i} + \sum_{j,j'\in[J]} q_{jj'} \cdot \underbrace{\nu_{jj'}^{\boldsymbol{\mu}}}_{\text{hub-hub static pricing problem}} .$$

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spoke-specific DP for i

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- With $\pi(\delta)$, spoke-hub requests are priced **dynamically** based on x_i:

Spoke i - Hub j requests: use $\underbrace{d_i(x_i, \mathsf{out}_j)}_{\text{from } i \text{ to } j}$ or $\underbrace{d_i(x_i, \mathsf{in}_j)}_{\text{from } j \text{ to } i}$.

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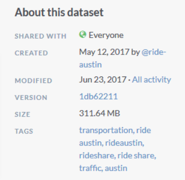
- Similar performance bounds when hubs are "uniformly related."
- Can incorporate spoke-spoke connections and relocation times into bounds and policies. detail

RideAustin example

- **RideAustin** a nonprofit ride-hailing company in Austin, Texas.
- **Dataset**: 1.5 million transitions over 10 months (2016.6 - 2017.4).
  $\Rightarrow$ Note: relocation times modeled (assumed deterministic)



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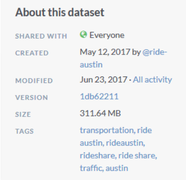

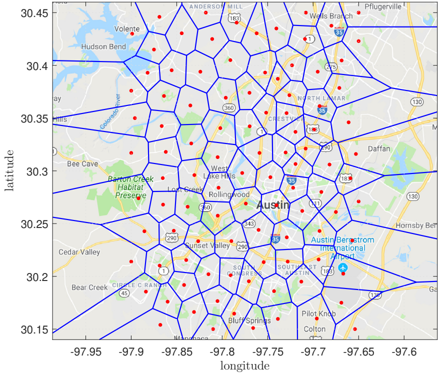

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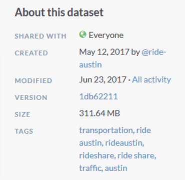
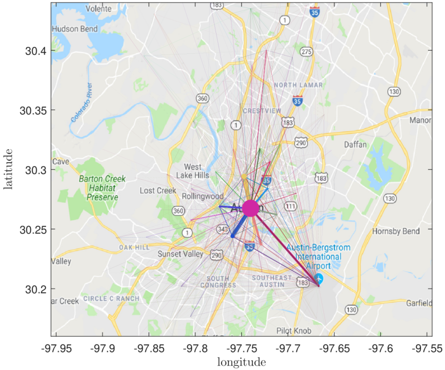

About this dataset
SHARED WITH Everyone
CREATED May 12, 2017 by @ride-austin
MODIFIED Jun 23, 2017 · All activity
VERSION 1db62211
SIZE 311.64 MB
TAGS transportation, ride austin, rideaustin, rideshare, ride share, traffic, austin



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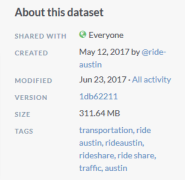
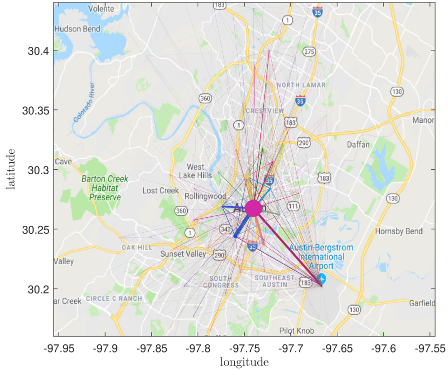
- **Challenge:** how do we choose the hubs? How many hubs?





Challenge: how to choose hubs

- Trade-off:
  - **Small number of hubs** $\Rightarrow$ retain benefits of dynamic pricing.
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Challenge: how to choose hubs

- Trade-off:
  - **Small number of hubs** $\Rightarrow$ retain benefits of dynamic pricing.
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- The approach:
  1. Fix number of hubs: select best hubs to maximize the flow covered by hubs (by solving an integer program).
  2. Choose optimal number of hubs by evaluating our Lagrangian bound and policy (incorporating travelling times and spoke-to-spoke transitions).

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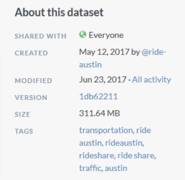

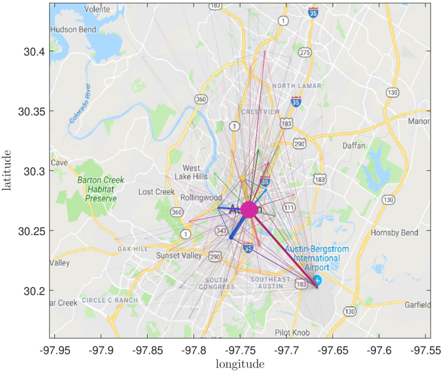

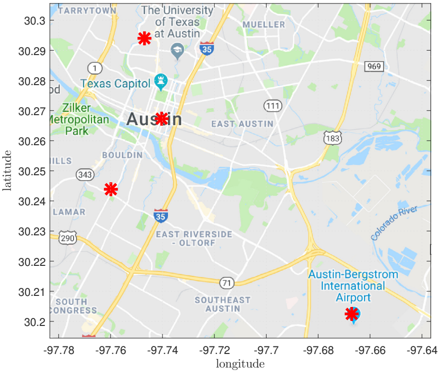

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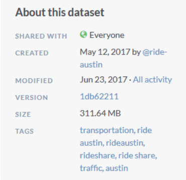

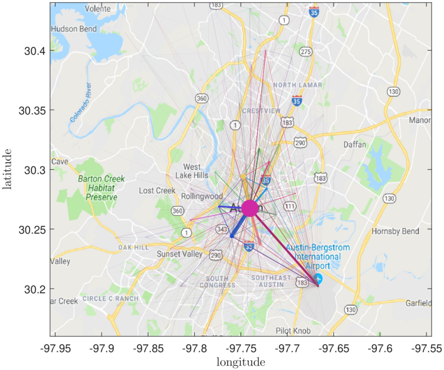

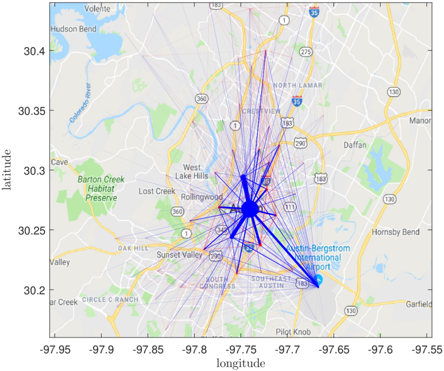

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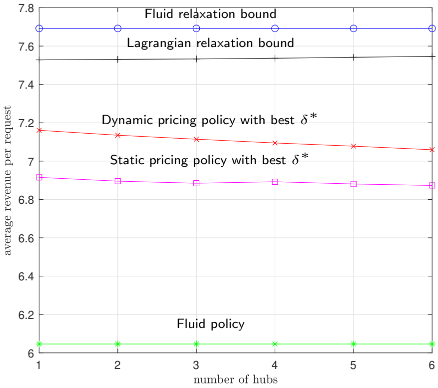

Performance gap

- Fluid policy: $\big(V^{\mathrm{F}} - V(\pi^{\mathrm{F}})\big)/V(\pi^{\mathrm{F}}) = \mathbf{27.22\%}$.
- Static pricing policy with $J = 1$: $\big(V^{\mathrm{R}} - V^{S}(\delta^*)\big)/V^{S}(\delta^*) = \mathbf{8.87\%}$. details
- Dynamic pricing policy with $J = 1$: $\big(V^{\mathrm{R}} - V^{\pi}(\delta^*)\big)/V^{\pi}(\delta^*) = \mathbf{5.13\%}$.

Takeaways and future directions

We study **dynamic pricing** of relocating resources in **large networks**.

- We develop performance bounds and policies based on Lagrangian relaxations.
  - **Hub-and-spoke networks:** policies are within $O(\sqrt{\ln n/n})$ of optimal.
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Reference: Balseiro, S.R., D.B. Brown, and C. Chen. 2019, "Dynamic pricing of relocating resources in large networks," *Management Science* (forthcoming).

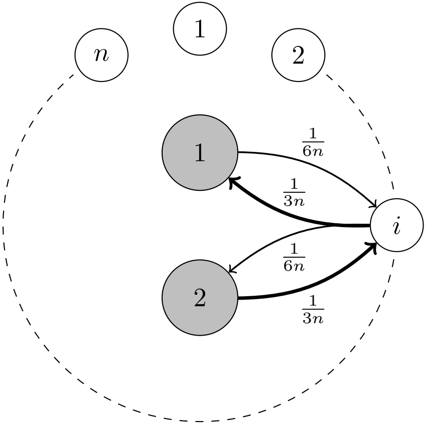
`https://papers.ssrn.com/abstract=3313737`

Two hub examples

(a) symmetric spokes with

$$\underbrace{q_{i1} = \frac{1}{3n},\ q_{1i} = \frac{1}{6n}}_{\text{hub 1}} \qquad \underbrace{q_{i2} = \frac{1}{6n},\ q_{2i} = \frac{1}{3n}}_{\text{hub 2}};$$

(b) $m = 2n$; all private values $\sim \mathrm{U}[0,1]$.



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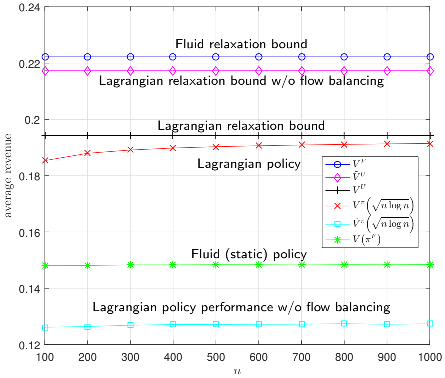

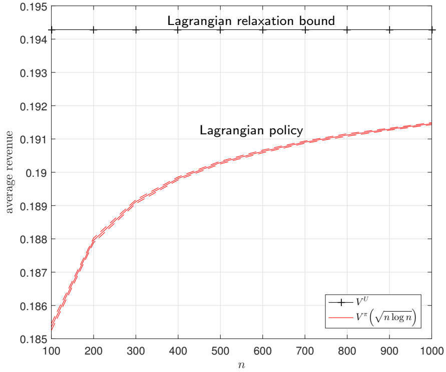

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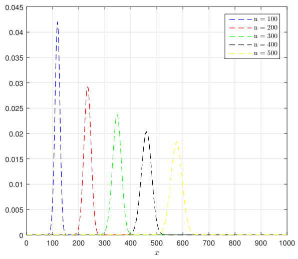
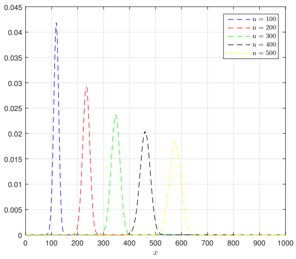
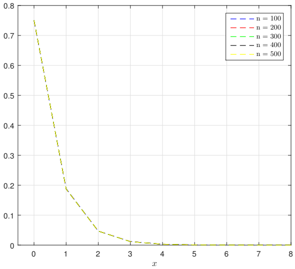


Hub distribution (total) without flow balance



Hub distribution (hub #1) without flow balance



Hub distribution (hub #2) without flow balance

Two hub examples

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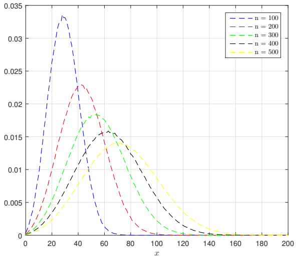

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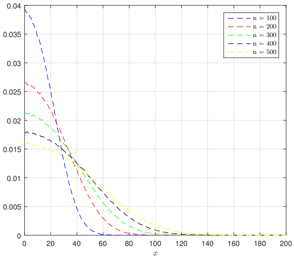

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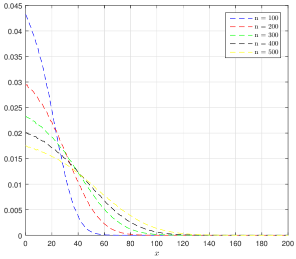

Hub distribution (hub #2) with flow balance

Optimizing over static policies

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fluid-based policies $\neq$ **static policies**

Optimizing over static policies

	flow from i to 0		flow from 0 to i
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How can we optimize over static policies?

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How can we optimize over static policies?

- Use the same relaxation, but restrict to static controls.
- The Lagrangian dual problem:

spoke-specific static pricing problem

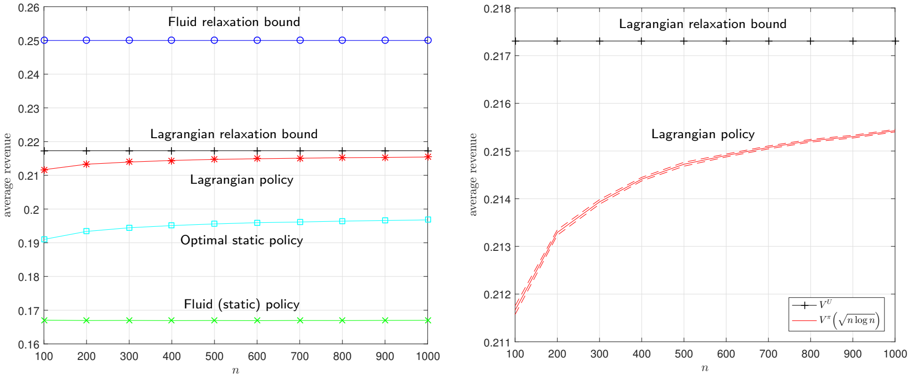
upper bound on best static policy

$$V^{\mathrm{S,R}} = \min_{\lambda \geq 0} \left\{ m\lambda + \sum_{i=1}^{n} h_i^{\mathrm{S}}(\lambda) \right\}$$

- Policy converges to optimal static policy in large network regime (by similar analysis).
- We can show that static policies are sometimes strictly suboptimal. back

Single hub examples revisited

(a) symmetric spokes with $q_{i0} = q_{0i} = \frac{1}{2n}$; (b) $\frac{m}{n} = 2$; (c) all private values $\sim \mathrm{U}[0,1]$



- Lagrangian policy is asymptotically optimal.
- Fluid policy: performance gap $\left(V^{\mathrm{F}} - V(\pi^{\mathrm{F}})\right)/V(\pi^{\mathrm{F}}) = \frac{2}{3}$.
- Optimal static policy converges to 0.2 (better than fluid but sub-optimal).

Performance analysis

Goal: $\underbrace{V^{\pi}(\delta)}_{\text{Lagrangian-based policy}} \leq V^{\mathrm{OPT}} \leq V^{\pi}(\delta) + \underbrace{\mathsf{Error}(n)}_{\xrightarrow[\frac{m}{n}\text{ fixed}]{n\to\infty} 0}$

back

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We know:

$$V^{\pi}(\delta) \leq V^{\text{OPT}} \leq V^{\text{R}}$$

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We know:

$$V^{\pi}(\delta) \leq V^{\mathrm{OPT}} \leq V^{\mathrm{R}} = \underbrace{\Big[V^{\mathrm{R}} - V^{\mathrm{R}}(\delta)\Big]}_{\textbf{Step (1)}} + \underbrace{\Big[V^{\mathrm{R}}(\delta) - V^{\pi}(\delta)\Big]}_{\textbf{Step (2)}} + V^{\pi}(\delta)$$

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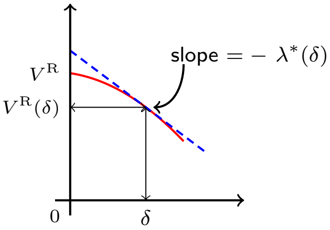
$$V^\pi(\delta) \leq V^{\mathrm{OPT}} \leq V^{\mathrm{R}} = \underbrace{\left[V^{\mathrm{R}} - V^{\mathrm{R}}(\delta)\right]}_{\textbf{Step (1)}} + \underbrace{\left[V^{\mathrm{R}}(\delta) - V^\pi(\delta)\right]}_{\textbf{Step (2)}} + V^\pi(\delta)$$

Step (1):

$V^{\mathrm{R}}(\delta)$ is generally not an upper bound for $\delta > 0$, but sensitivity analysis yields:



$$V^{\mathrm{R}}(\delta) \leq V^{\mathrm{R}}(0) = V^{\mathrm{R}} \leq V^{\mathrm{R}}(\delta) + \lambda^*(\delta) \cdot \delta$$

Performance analysis

Goal: $\underbrace{V^{\pi}(\delta)}_{\text{Lagrangian-based policy}} \leq V^{\mathrm{OPT}} \leq V^{\pi}(\delta) + \underbrace{\mathsf{Error}(n)}_{\xrightarrow[\frac{m}{n}\text{ fixed}]{n\to\infty} 0}$ back

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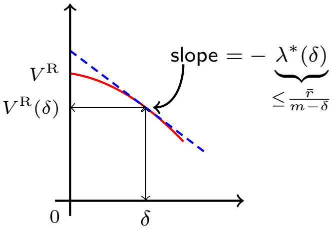

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$$\boxed{V^{\mathrm{R}} - V^{\mathrm{R}}(\delta) \leq \bar{r} \cdot \frac{\delta}{m - \delta}}$$

Performance analysis

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Step (2):

- $V^{\pi}(\delta)$: performance of Lagrangian policy in **original system** (hub **cannot** hold negative number of resources)

- $V^{\mathrm{R}}(\delta)$: performance of Lagrangian policy in **relaxed system** (hub **can** hold negative number of resources)

Performance analysis

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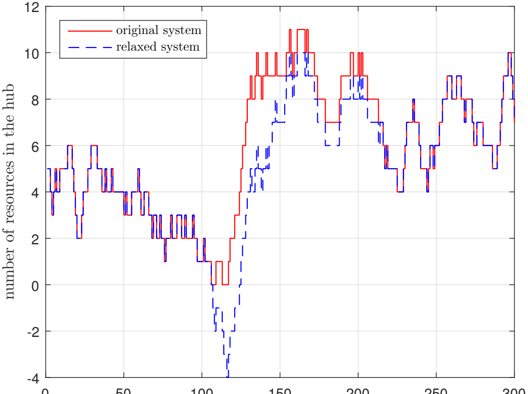

Performance analysis

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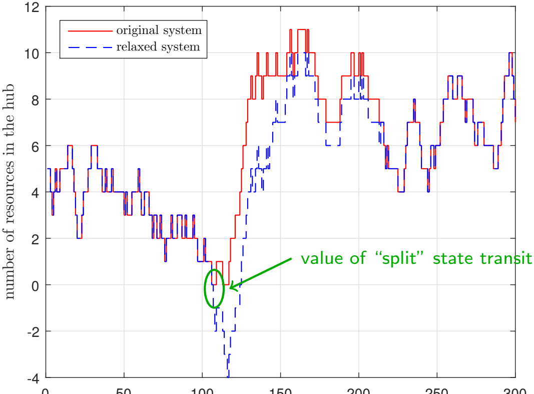

value of "split" state transition $\leq \bar{r} + \bar{\omega}$

upper bound on derivative of $r(d)$

Performance analysis

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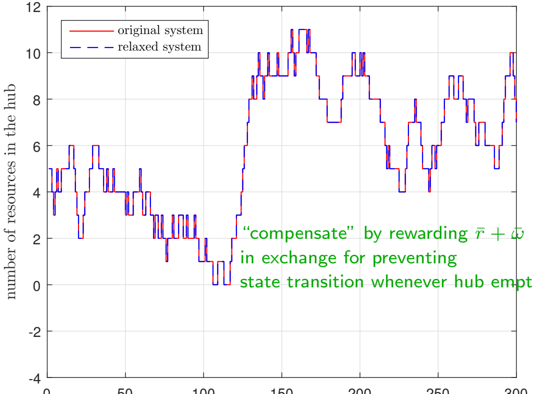

“compensate” by rewarding $\bar{r} + \bar{\omega}$
in exchange for preventing
state transition whenever hub empties

Performance analysis

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Step (2):

By comparing the value functions in the relaxed and original systems:

$$\boxed{V^{\mathrm{R}}(\delta) - V^{\pi}(\delta) \leq (\bar{r} + \bar{\omega}) \cdot \mathbb{P}\left[X_0(\delta) = 0\right]}$$

upper bound on derivative of $r(d)$

hub depletion probability with $\pi(\delta)$

Performance analysis

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Step (1): $V^{\text{R}} - V^{\text{R}}(\delta) \leq \bar{r} \cdot \frac{\delta}{m-\delta}$.

Step (2): $V^{\text{R}}(\delta) - V^{\pi}(\delta) \leq (\bar{r} + \bar{\omega}) \cdot \mathbb{P}\big[X_0(\delta) = 0\big]$.

Proposition. *The Lagrangian policy* $\pi(\delta)$ *satisfies*

$$V^{\pi}(\delta) \leq V^{\text{OPT}} \leq V^{\text{R}} \leq V^{\pi}(\delta) + \underbrace{\bar{r} \cdot \frac{\delta}{m-\delta}}_{\text{set } \delta \text{ small!}} + \underbrace{(\bar{r} + \bar{\omega}) \cdot \mathbb{P}\Big[X_0(\delta) = 0\Big]}_{\text{set } \delta \text{ big!}}.$$

Incorporating Spoke-spoke Connections and Relocation Times

Spoke-spoke requests: relax the **relocation constraint** at destination spoke:

$$(i, i') \ : \ \underbrace{x_{i',t+1} = x_{i',t} + \ Z_i}_{\text{Lagrange mult. } \nu_{i,i'}} \longrightarrow \sim\text{Bernoulli}(d_i)$$

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Positive relocation times:

1. same relaxations to decompose over spokes.
2. enable resources moving to the spoke to be instantaneously available at the spoke.
3. only need to track the number of resources in the spoke (use Little's law for resources leaving the spoke).

back