Incentivizing Resource Pooling

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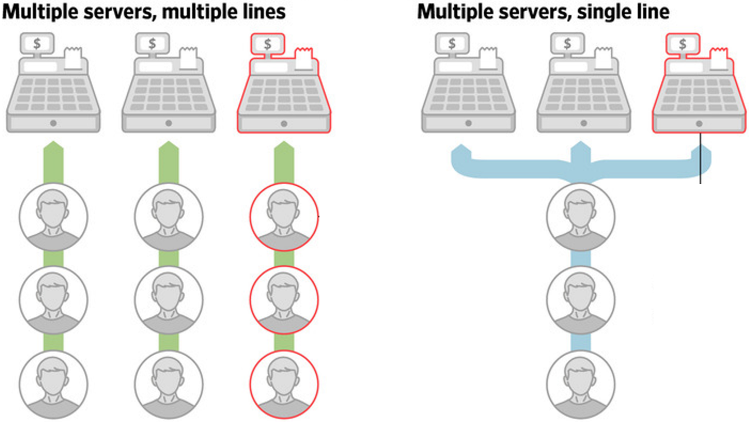
The Line Dance

Queuing theory, the mathematical study of lines, helps businesses, call centers, computer networks and others figure out how to keep things moving.

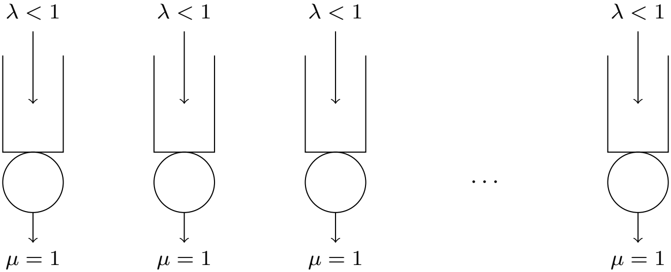


THE WALL STREET JOURNAL.

Resource pooling significantly improves service

Resource pooling: known fact

N servers: job arrival rate $\lambda < 1$, server processing rate $\mu = 1$



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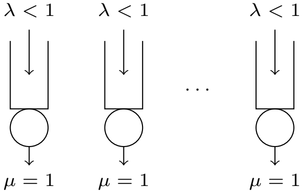

$vs.$

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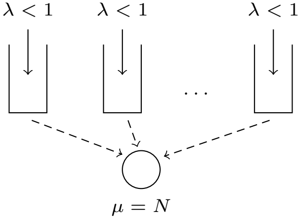

 # jobs in system: $N \cdot \frac{\lambda}{1-\lambda}$ (linear) vs. $\frac{\lambda}{1-\lambda}$ (constant)

Can resource pooling be achieved in **decentralized** systems?

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Decentralization boosts security, privacy, and scalability

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Golem Network	Akash Network	iExec
Market cap: \$170M	Market cap: \$320M	Market cap: \$75M

- Essential aspects of the problem:
  - Large-scale system: number of servers N is large
  - Servers have limited information about one another

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  - Provides tight theoretical guarantees

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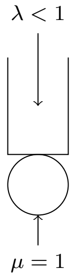
- Propose an approximation-based analytical framework
  - Simplifies the design and analysis of token-based mechanisms
  - Provides tight theoretical guarantees
  - Can be applied to more general settings

Model setup

- N strategic servers
- Jobs arrive with Poisson(λ) where $\lambda < 1$; capacity units arrive with Poisson(1)



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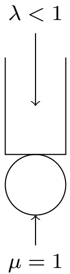

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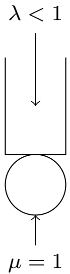

- **Servers' objective:** minimizing own long-run average total cost
- Limited information:
  (a) A server's arrivals and actions are private information
  (b) Precise knowledge of number of servers N not required (except knowing that it is relative large)

Related Literature

Resource pooling:

- Power of resource pooling: [Tsitsiklis and Xu, 2013]
- Decentralized setup with two servers: [Hu and Caldentey, 2023]

Mean-field equilibrium:

- Analysis of complex operational problems: [Iyer et al., 2014], [Balseiro et al., 2015], [Kanoria and Saban, 2021], [Arnosti et al., 2021]
- Fluid mean-field equilibrium similar in spirit to [Balseiro et al., 2015]

Scrip system:

- Analysis of scrip system: [Kash et al., 2007], [Kash et al., 2015], [Johnson et al., 2014], [Bo et al., 2018]

Other related work:

- Cooperative game model: [Anily and Haviv, 2010], [Anily and Haviv, 2014], [Karsten et al., 2015]
- Supermarket game: [Xu and Hajek, 2013], [Yang et al., 2019]

Outline

- **Motivation, research question, and literature review**
- **Token-based mechanism**
  - Solution concept: Fluid mean-field equilibrium (FMFE)
  - Characterization of FMFE
  - Designing key element of mechanism
- **FMFE strategy as near-optimal best response**
  - Asymptotic analysis for large markets
  - Numerical analysis for small markets
- **Extension to heterogeneous servers**
- **Takeaway**

Token-based mechanism

In the mechanism, a server can:

- Request help from others <u>without recall</u> at any time.

- When a capacity unit arrives, either: (i) serve its job, (ii) help others, or (iii) be idle and waste the unit <u>without recall</u>.

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  - The *oldest* job in shared pool is served (if pool is non-empty)
- A **shared pool** to match requests and provisions of help in FCFS order.

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- The value of ϕ is critical to system performance

Equilibrium concept: Fluid mean-field equilibrium

Approximation methodology similar to (Balseiro et al. 2015)

- **Mean-field approximation:** each server optimizes by assuming state of shared pool is fixed at long-run average $\implies$
  - Expected waiting time in shared pool is constant $w \geq 0$: value determined endogenously by equilibrium
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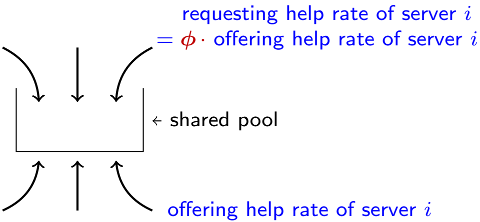

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  $\implies$ Closed-form characterization (next slide)
- Fluid mean-field equilibrium (FMFE):

$$\boxed{w \in H(w)}$$

conjectured waiting time

induced waiting time when all servers follow best responses to w

Server's best response

Closed-form solution: threshold policy w.r.t. queue length:

- Request help only when queue length exceeds a threshold k (which depends on ϕ and w)
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Proof: Using a drift analysis.

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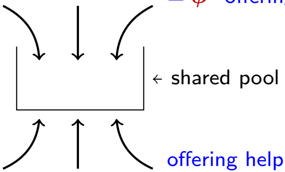

Best response when $w < 1$

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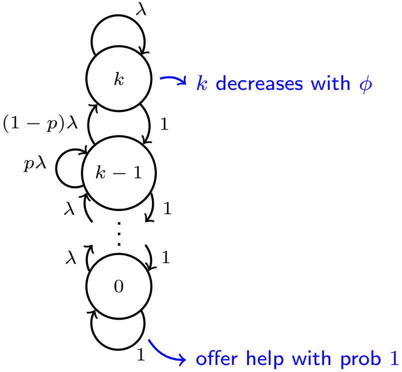

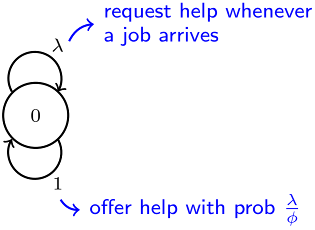

$\boldsymbol{\phi \leq \lambda}$ $\qquad$ $\boldsymbol{\phi \geq \lambda}$

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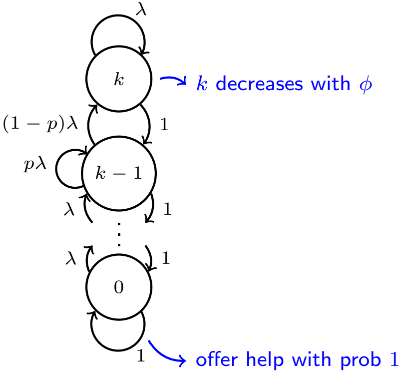

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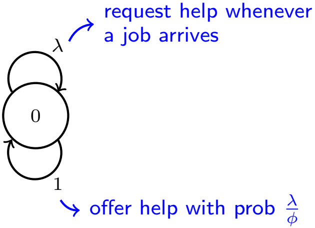

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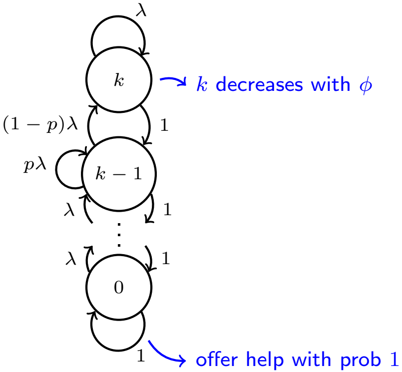

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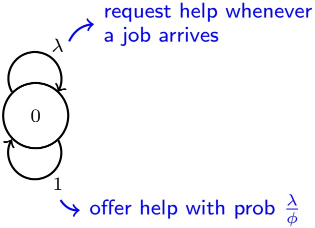

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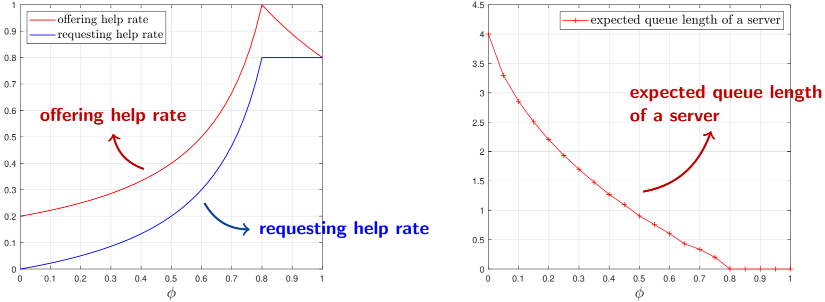
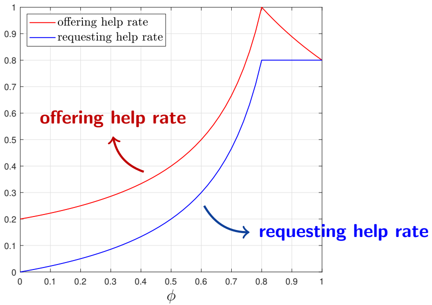
Optimal value of ϕ

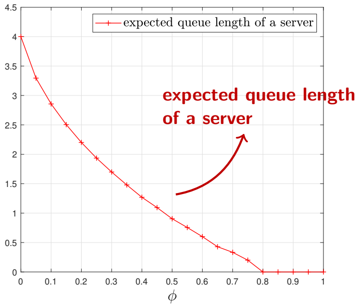


Proposition. The expected total number of jobs in system, denoted by $Q_\Sigma(\phi)$, satisfies:

1. When $\phi < \lambda$: $\lim_{N\to\infty} Q_\Sigma(\phi)/N = q(\phi) > 0$
2. When $\phi \geq \lambda$: $Q_\Sigma(\phi) = \frac{\phi}{1-\phi}$

Optimal value of ϕ





Main result. The optimal value is $\boldsymbol{\phi = \lambda}$. Moreover, this induces complete resource pooling: it is each server's best strategy to (i) request help whenever a job arrives, (ii) offer help when queue is empty.

$\Longrightarrow$ System's dynamics and performance match those under centralized control

FMFE as **good approximation** of servers' strategics

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2. A coupling argument and a drift analysis to show:
   (a) shared pool's queue length transitions to stationary distribution quickly as $N \to \infty$
   (b) in stationary distribution, shared pool is non-empty with probability $\phi - \frac{c(\lambda,\phi,\delta)}{N^{1-\delta}}$

Analysis for small market

- Mechanism uses $\phi = \lambda$.
- Consider the fluid setup: tokens can go negative but expected rates of earning and spending tokens are equal.
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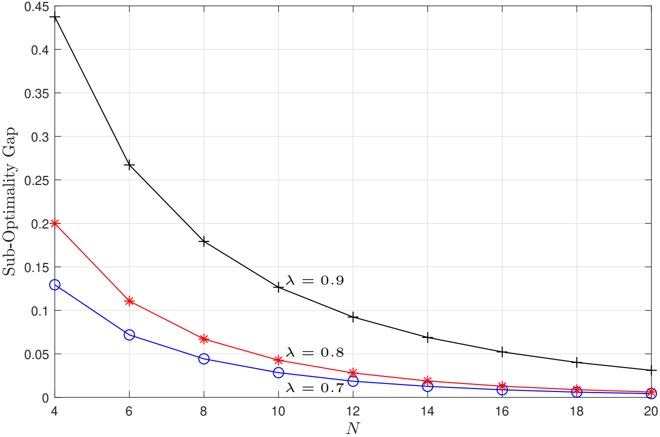
$\Rightarrow$ Optimal strategy depends only on two states: q_1 (own queue length) and q_0

$\Longrightarrow$ Tractable optimization problem!

Numerical results

(a) job processing cost $c = 1$; (b) job arrival rate $\lambda \in \{0.7, 0.8, 0.9\}$

Sub-optimality gap $= \frac{\text{Cost of complete resource pooling} - \text{Cost of optimal strategy}}{\text{Cost of optimal strategy}}$



- The value of playing strategically is small even with few servers (and when server one can perfectly monitor the shared pool)

Extension: heterogeneous servers

- For each server i: job arrival rate λ_i and processing rate μ_i; let $\rho_i = \frac{\lambda_i}{\mu_i}$
- Assume $0 < \underline{\rho} \leq \rho_i \leq \bar{\rho} < 1$ and $0 < \underline{\lambda} \leq \lambda_i \leq \bar{\lambda}$ for all servers

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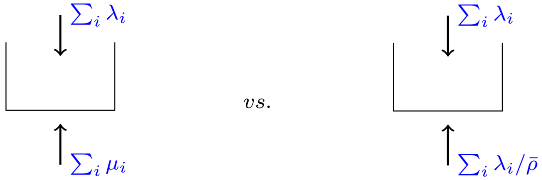

Extension: heterogeneous servers

- For each server i: job arrival rate λ_i and processing rate μ_i; let $\rho_i = \frac{\lambda_i}{\mu_i}$
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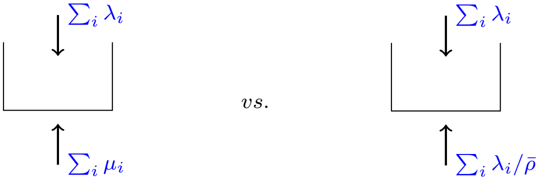

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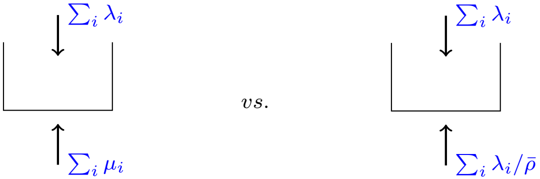

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- Job processing costs are allocated $\propto \mu_i$ versus $\propto \lambda_i$

  $\Rightarrow$ Costs allocated fairly in our mechanism!

Summary

- We study incentivizing resource pooling in a decentralized setting, where servers have limited information about others

- Operational takeaway: A simple **token-based mechanism** incentivizes complete resource pooling when number of servers is large
  - Analysis based on fluid mean-field equilibrium
  - Numerical results show that benefit from unilateral deviation is small even with only a few servers

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Reference: C. Chen, Y. Chen, and P. Qian. 2023. Incentivizing Resource Pooling. Under review.

Working paper available at `https://papers.ssrn.com/abstract=4586771`

Appendix

Minimum number of servers to sustain FMFE

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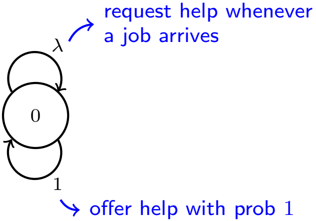

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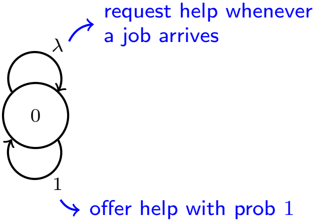
  - $w \leq 1 \Rightarrow N \geq \left\lceil \frac{1}{1-\lambda} \right\rceil$



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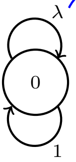
request help whenever
a job arrives



offer help with prob 1

back