Optimal Incentive Design for Decentralized Dynamic Matching Markets

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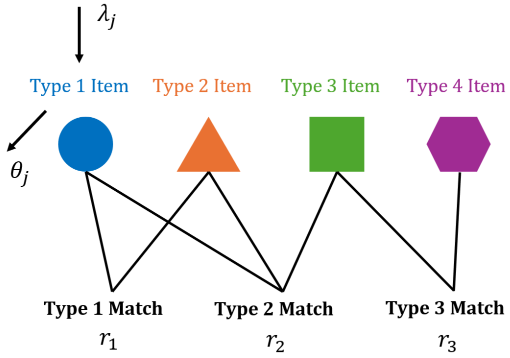
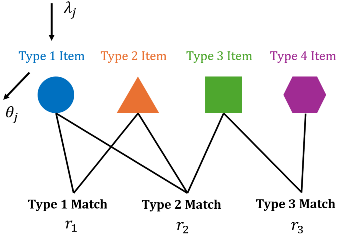
Dynamic matching



- Heterogeneous item and match types
- Stochastic arrivals and departures with rates λ_j and θ_j

Dynamic matching



λ_j

Type 1 Item Type 2 Item Type 3 Item Type 4 Item

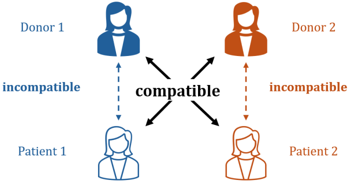
θ_j

Type 1 Match r_1

Type 2 Match r_2

Type 3 Match r_3

Examples: kidney exchange



Donor 1

Donor 2

incompatible

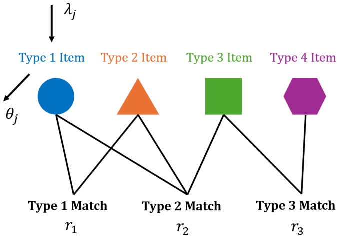
compatible

incompatible

Patient 1

Patient 2

Dynamic matching



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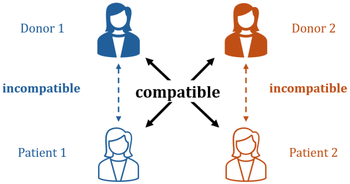
Examples: kidney exchange (multi-way matches possible)



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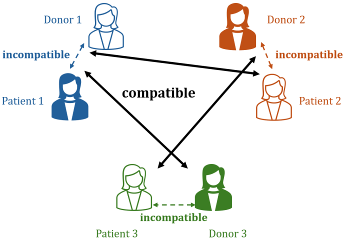

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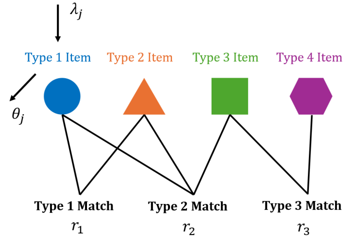
Donor 3

Dynamic matching



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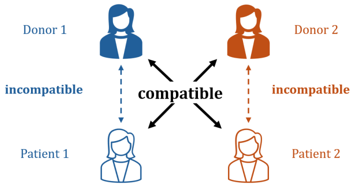
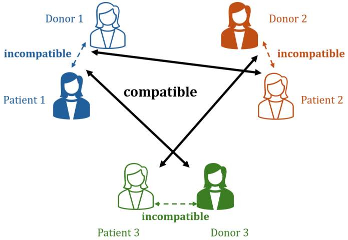


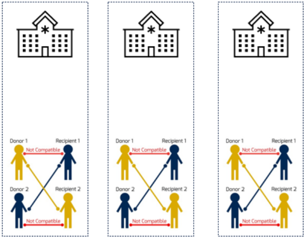


- **Decisions:** when and how to match
- **Objective:** maximize long-run average match values

Decentralized dynamic matching

We focus on *decentralized* setting:

- Example: multi-hospital kidney exchange



- Agents (e.g., hospitals) each manage own streams of items

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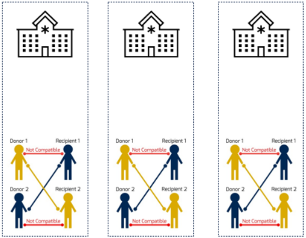
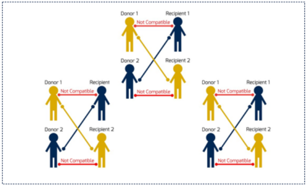


$vs.$

NATIONAL KIDNEY REGISTRY®
FACILITATING LIVING DONOR TRANSPLANTS



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thicker market = better matches

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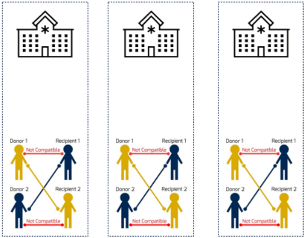

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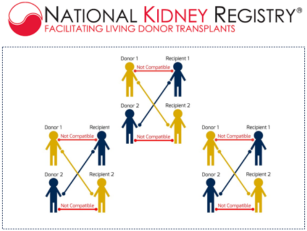

- Agents (e.g., hospitals) each manage own streams of items
- **Goal:** mechanism to incentivize full participation ⇒ maximizes social efficiency
- **Challenges** beyond algorithm design: handling strategic behaviors

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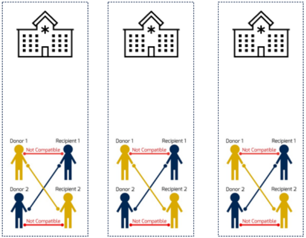

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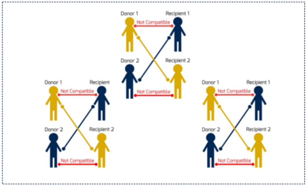
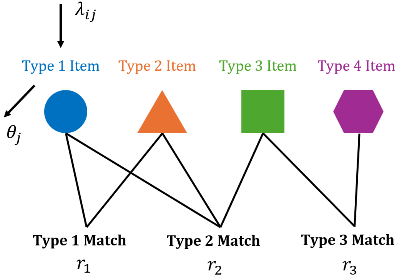


- **Main result**: develop simple and intuitive monetary and non-monetary mechanisms that incentivize complete item submissions when # agents is large.

  $\Longrightarrow$ System dynamics and performance match those under centralized control in large markets

Model setup

- N strategic agents: Each agent i manages a local dynamic matching
  - Type-j items arrive at rate λ_{ij} and depart at rate θ_j
  - Performing type-m match generates reward r_m
- **Agents' objective:** maximizing own long-run average reward



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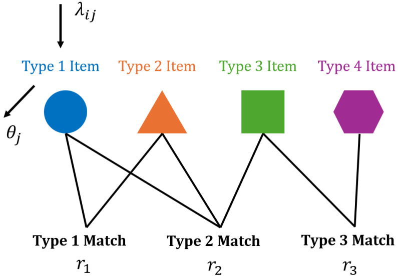
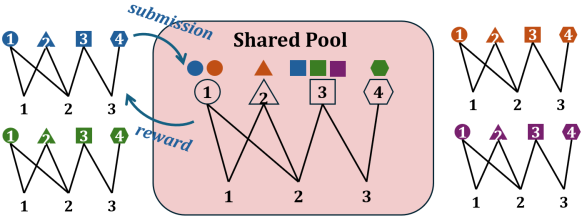
- Private information: item arrivals and actions not observable by others



Model setup: Market design

- Additional agent action: submission
- Mechanism design: how to reward submission to incentivize full submission



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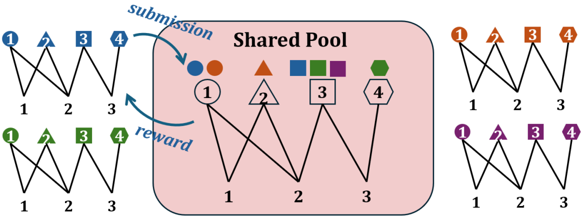

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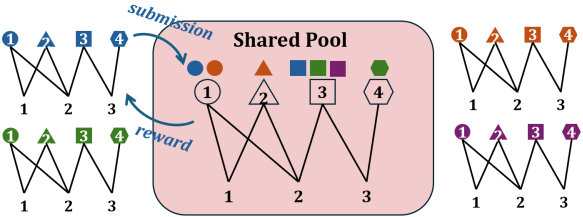
- Centralized matching at shared pool
- Huge design space: reward can depend on item type, matching outcome, and can be monetary or non-monetary (e.g, priority, credits)



Benchmark: centralized control

Fluid relaxation of centralized problem:

$$\begin{aligned} \max_{x_m \geq 0} \quad & \sum_{m=1}^{K} r_m x_m \\ \text{s.t.} \quad & \sum_{m=1}^{K} M_{jm} x_m \leq \lambda_j, \forall\, j \leq J. \end{aligned}$$

$\lambda_j = \sum_{i=1}^{N} \lambda_{ij}$: aggregate item arrival rate

(capacity constr.)

number of type-j items in match m

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x_m^*: **matching rate** of match m

- $x_m^* > 0$: m is essential match
- $x_m^* = 0$: m is non-essential match

Dual:

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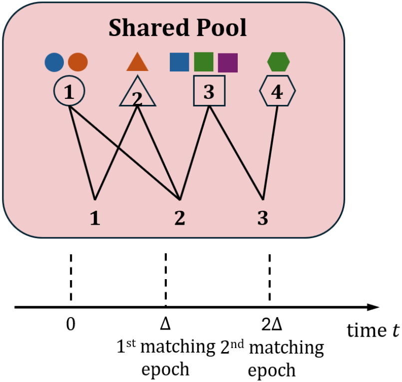
- Centralized matching at shared pool: Any algorithm that is (i) asymptotically optimal as $N \to \infty$ and (ii) restricted to $\mathcal{M}_+$

Centralized matching at shared pool

Example: Periodic matching policy

- Shared pool performs matching every $\Delta = o(1)$ (e.g., $\Delta = N^{-1/3}$)
- Performance of our mechanism depends on regret of centralized matching relative to fluid bound



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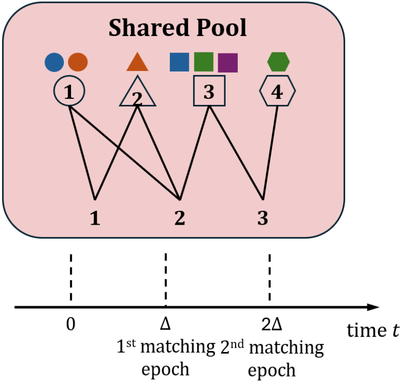
How to incentivize full item submission?



Incentive design

- We propose two designs that are intuitive and easy to implement:
  - **Marginal-Value (MV)** mechanism
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- Theoretical guarantees:
  - Full submission is approximate Nash equilibrium under MV
  - Full submission guarantees a stronger mean-field equilibrium under MVC

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Key observation: reimbursing each submitted item using its marginal value $p_j^* \Rightarrow$ full submission is dominant strategy

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Fluid relaxation of agent i's problem:

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submission rate of type-j items

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$\Longrightarrow$ For *any* agent, full submission is optimal

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Intuition: over-demanded jobs get matched with probability one as $N \to \infty$, which ensures an expected payoff of p_j^*.

Marginal-Value (MV) mechanism

Let us examine the incentive structure:

- Favorable incentives for over-demanded items: when matched with high probability in shared pool (i.e., when N is large)
  - Holding is strictly suboptimal
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$\Longrightarrow$ the MVC mechanism!

Equilibrium concept: Mean-field equilibrium

A stronger equilibrium concept than approximate Nash equilibrium

- **Mean-field approximation:** agents assume the shared pool is always in steady state $\implies$
  - Probability that a submitted type-j item is matched is constant $w_j \in (0, 1)$, independent of history and determined endogenously

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- **Mean-field equilibrium (MFE):**

$$\boxed{\boldsymbol{w} = H(\boldsymbol{w})}$$

conjectured matching probs

induced matching probs when all agents follow best responses to $\boldsymbol{w}$

Marginal-Value-plus-Credit (MVC) mechanism

- **Observation:** submissions of under-demanded items need to be rewarded, even though their marginal value is zero.
- Simply tweaking the allocation probs in MV mechanism does *not* make full submission an MFE.

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- Over-demanded: ●
- Under-demanded: ▲ ⬢

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▲	△
⬣	⬡

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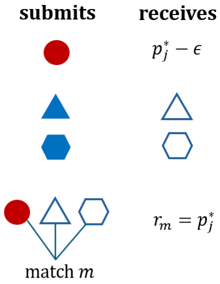
- Submitting a single under-demanded type-j item yields a type-j credit, which departs at same rate.
- Submitting an over-demanded type-j item, when paired with under-demanded credits that forms a "virtual match" $m \in \mathcal{M}_+$, yields payoff $r_m = p_j^*$ when matched.



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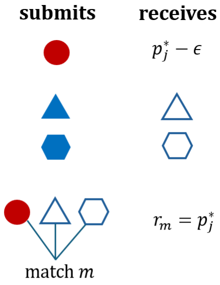
Marginal-Value-plus-Credit (MVC) mechanism

MVC mechanism (a refinement of the MV mechanism):

- Submitting a single over-demanded type-j item yields payoff $p_j^* - \epsilon$ (small tax) when matched.
- Submitting a single under-demanded type-j item yields a type-j credit, which departs at same rate.
- Submitting an over-demanded type-j item, when paired with under-demanded credits that forms a "virtual match" $m \in \mathcal{M}_+$, yields payoff $r_m = p_j^*$ when matched.
- Expired credits convert into one-time lotteries for collected taxes ϵ.



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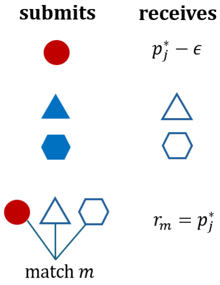
- Mechanism can be implemented without money via randomized matching allocations.



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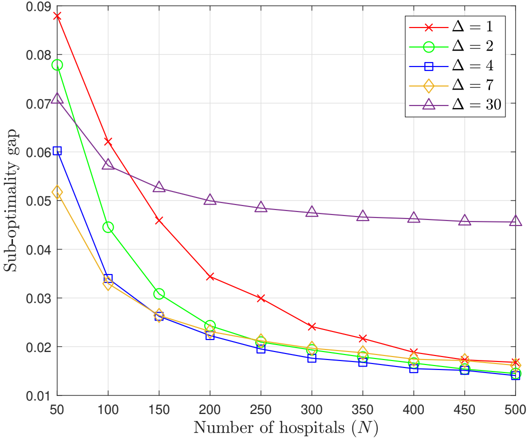
Marginal-Value-plus-Credit (MVC) mechanism

Theorem. Under the MVC mechanism, full item submission by all agents constitutes an MFE when $N \geq N_0$ (some constant) and also an approximate Nash equilibrium.

Numerical results

Multi-hospital kidney exchange example based on real data

Sub-optimality gap $= \frac{\text{Fluid relaxation bound} - \text{Payoff from full submission}}{\text{Payoff from full submission}}$



Summary

- We study optimal incentive design in decentralized dynamic matching, where agents have limited information about others (so deviation cannot be punished directly)

- **Operational takeaway:** simple marginal-value based mechanisms incentivize **full** item submission in large markets
    - MV mechanism: full submission is approximate Nash equilibrium
    - MVC mechanism: full submission is a stronger mean-field equilibrium

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Reference: C. Chen, P. Qian, and J. Zhang. 2024. Optimal Incentive Design for Decentralized Dynamic Matching Markets. Major Revision at *MS*.

Appendix